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TITLE XUV HARMONIC ENHANCEMENT BY MAGNETIC FIELDS

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XUV Harmonic Enhancement by Magnetic Fields

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1. Introduction We examine three ways to enhance harmonic output [1,2] of an XUV planar free-electron laser (FEL) operating in the Compton regime [3]. The first method is to increase the rms static magnetic field, making it as large as possible. The second is by adding effective magnetic fields at the harmonics, thereby increasing the coupling to the harmonics. The third is by phase programming; i.e. programming the magnetic field to introduce jumps in the phase of the electrons as they move through phase space.

The important concept in dealing with harmonics is the effective field. When the magnetic field is weak, the effective field on the fundamental is just the amplitude of the actual magnetic field. But when the field increases, the axial component of motion of an electron in the magnetic field begins to have a significant variation from steady motion. This first appears in the form of a sinusoidal variation in the time versus distance plot with a period that is half a wiggler period. Thus, the motion of an electron that is sinusoidal in s is not strictly sinusoidal in t, and higher (odd) harmonics are generated by this accelerating electron. Analysis of the motion of the electron shows that when it interacts with an electromagnetic wave, it behaves as if the field on the fundamental were reduced, and this reduced field we call the effective field. Likewise, on the odd harmonics, even though there is no static field that would give rise to harmonics directly at low magnetic field, the non-steady s motion generates fields as if there were a finite field, again we call this field, the effective field at the harmonic. It is well known that a sinusoidal magnetic field generates effective fields at the harmonics that are obtained by multiplying the fundamental magnetic field amplitude by the difference of two Bessel functions. Here we enhance these non-linear effective fields by the addition of small amounts of the harmonic fields, to create new values of the effective field.

In addition to these effective fields, we can also enhance the harmonic content by phase programming the magnetic field. The demonstration of this principle is shown in an example where we have achieved early saturation on the third harmonic, well before the fundamental has saturated. In this case, an insertion of a

π phase shift to the third harmonic ($\pi/3$ to the f· damental) at its saturation peak causes the downward turning curve to continue its upward course. In this example little effect occurs to the fundamental.

A 1-D model of the harmonic generation process is sufficient to describe these processes. It allows for both the concept of an effective field and exhibits the phase programming phenomena. Other work has shown how some two dimensional effects can be incorporated into a 1-D model [4], and the ability makes the applicability of this model even greater than it would first appear. By a 1-D treatment, we assume that the phase fronts of the optical field are flat and that they interact with an electron beam whose electrons have no variation in transverse properties. We further assume for the purposes of this paper that the filling factor [5] is unity, i.e. each unit area of the electron beam radiates into a unit area of the optical beam. This gives a simple model in which to study these effects.

2. Mathematical Description

The wiggler contains a magnetic field $B(Z)\hat{e}_x$ that is periodic

$$B(Z) = B(Z + 2\pi/k_w). \tag{1}$$

This field gives rise to a static vector potential $A(Z)\hat{e}_y$ where

$$\vec{B} = \vec{\nabla} \times \vec{A}, \tag{2}$$

or

$$B(Z) = -A'(Z). \tag{3}$$

Conservation of transverse momentum equates the sum of the mechanical momentum $m\gamma v_y$ and the field momentum $-eA$ (MKSA) to the total canonical momentum, P_y,

$$P_y = m\gamma v_y - eA, \tag{4}$$

where e is the magnitude of an electron charge, $m\gamma c^2$ is the total (relativistic) energy of the electron, v_y is the electron velocity in the y direction, and c is the speed of light. In accordance with our 1-D model, we take $P_y = 0$. This gives

$$v_y = \frac{ca_w}{\gamma}, \tag{5}$$

where

$$a_w = eA/mc. \tag{6}$$

When this electron is subjected to the 1-D electric field $E_y\hat{e}_y$, the total energy of the electron can change. The rate of change of total energy of the electron is obtained by dotting the Lorentz force equation with $\vec{v}$

$$\frac{dmc^2\gamma}{dt} = -ec\frac{a_w(k_w Z)}{\gamma}E_y. \tag{7}$$

This equation is averaged over a wiggler period where the periodic variations in the phase ϕ give rise to the effective fields [1,2] in the usual way, and we obtain following that notation [1]

$$\frac{d\gamma}{dt} = -c\Re\sum\frac{K_l}{2}\mathcal{E}_l\frac{\exp(il\phi)}{\gamma}, \tag{8}$$

where for simplicity we have taken $\mathcal{E}_l$ to be the scaled harmonic field component, i.e.

$$E_y = (mc^2/e)\sum\mathcal{E}_l\exp[il(k_s Z - \omega_s t)]. \tag{9}$$

At high γ these effective fields are given by a generalisation of the expression reported in [1],

$$K_l = 2\left\langle a_w\exp[-i\omega_s T_c(Z,\gamma) - ilk_w Z]\right\rangle, \tag{10}$$

where T_c is the periodic part of the time it takes a particle to reach location Z. The angle brackets indicate an average over one wiggler period. Here when a_w is a weak cosine field of amplitude a_c, K_l is normalised to have $K_1 = a_c$. Now T_c is given in terms of T by

$$\frac{dT}{dZ} = \frac{1}{\sqrt{v^2 - v_y^2}}, \tag{11}$$

where v is the magnitude of the velocity. At large γ values v_y is small compared to v and we obtain

$$\frac{dT}{dZ} \approx \frac{1 + \frac{1}{2}\gamma^{-2}a_w^2(k_w Z)}{v}, \tag{12}$$

and, by subtracting off the linear part, we obtain

$$T_c(Z) = T(Z) - (k_w/2\pi)T(2\pi/k_w)Z. \tag{13}$$

We are looking for an algorithm that is simple computationally. Such an alogrithm can be generated by using scaled variables. We transform to the dimensionless variables

$$\tau = \omega_s T, \tag{14}$$

$$\tau_c = \omega_s T_c, \tag{15}$$

$$z = k_w Z, \tag{16}$$

$$\xi \equiv \frac{k_s < a_w^2(k_w Z) >}{4lk_w \gamma^2} \approx \frac{< a_w^2 >}{2[1+ < a_w^2 >]} = \frac{\sum_1^\infty a_l^2}{4 + 2\sum_1^\infty a_l^2}, \tag{17}$$

and

$$a(z) = a_w(z) / < a_w^2(z) >^{1/2}. \tag{18}$$

In eq.(17) we have written the expression for ξ with the Fourier components a_l of a_w. Using dimensionless variables and integrating, eq.(12) becomes

$$\tau(z) = \frac{\omega_s}{k_w v} z + 2l\xi D(z), \tag{19}$$

where

$$D(z) = \int_0^z a^2(z) dz. \tag{20}$$

We can then compute

$$\tau_c(z) = \tau(z) - \tau(2\pi) z/(2\pi), \tag{21}$$

which when combined with eq.(19) gives

$$\tau_c(z) = 2l\xi[D(z) - D(2\pi) z/(2\pi)]. \tag{22}$$

Combining eq..(10) and (13)-(22), we obtain

$$K_l = \sqrt{\frac{8\xi}{1-2\xi}} \langle a(z) \exp\{-il[2\xi D(z) + z - 2\xi D(2\pi) z/(2\pi)]\} \rangle. \tag{23}$$

In order to use the equations in a computer code, it is convenient to have flexibility in the input. This is accomplished by inputing a magnetic field profile, whose absolute magnitude is arbitrary. This magnetic field, $b(z)$ integrates to the unscaled vector potential $a_0(z)$. The integral of the square of this latter quantity we refer to as $D_0(z)$, and it is the unscaled version of $D(z)$. The root-mean squared average of a_0 is just the square root of, the terminal value of D_0 divided by the terminal ordinate. We then have the following quantities

$$a_0'(z) = -b(z), \tag{24}$$

$$D_0'(z) = a_0^2(z) \tag{25},$$

$$\alpha = 2\pi/D_0(2\pi), \tag{26}$$

$$a(z) = \sqrt{\alpha}a_0(z), \tag{27}$$

and

$$D(z) = \alpha D_0(z). \tag{28}$$

With these expressions, we can integrate to get K_l as

$$K_l(\xi) = \frac{1}{2\pi}\int_0^{2\pi} dz\sqrt{\frac{8\xi\alpha}{1-2\xi}}a_0(z)\exp\{-il[2\alpha\xi D_0(z) + (1-2\xi)z]\}. \tag{29}$$

This expression for K_l depends only on the unnormalized profile $b(z)$ defined from 0 to 2π, and the parameter ξ a measure of the strength of the rms magnetic field.

In addition to the effective field K_l, it is also of interest to introduce the coupling efficiency k_l where

$$k_l(\xi) = \frac{K_l(\xi)}{< 2a_w^2 >^{1/2}} = K_l(\xi)\sqrt{\frac{1-2\xi}{4\xi}}. \tag{30}$$

This definition for k_l is normalised with unity modulus for weak sinusoidal magnetic fields.

3. The Enhancement Schemes

3.1. The rms Field

The first enhancement scheme is to increase the rms magnetic field. The rms magnetic field is a direct measure of the ξ parameter defined in eq.(17), and vice versa. Thus, as we increase ξ we increase the rms field and a resulting change occurs to the effective field on the l'th harmonic that depends on the magnetic field. In fig.(1) we show the effective filed plotted against ξ for a pure sinusoidal field. Here the coupling efficiency is well known and $k_l = |J_{(l-1)/2)}(l\xi) - J_{(l+1)/2}(l\xi)|$ for the l'th harmonic, and this combined with eq.(30) gives the effective field plotted in fig.(1). This graph illustrates that a good way to increase the effective field is to increase the rms field or ξ. There are, however, two caveats. The first is that there are physical limits as to how big ξ can be made because remanent fields of permanent magnetics are limited. The second is that as ξ increases, the resonant condition changes. In particular, if we have a third harmonic field at $\xi = 0.25$, it is at a wavelength that is 2/3 rather than 1/3 of the $\xi = 0$ fundamental.

3.2. Distorting the Sinusoidal Field

The second enhancement scheme is to add higher harmonic static magnetic fields. Another way of looking at this approach is to consider general periodic magnetic fields, not restricted to pure sinusoids. A special limiting case of eq..(17),(23), and (30) as $\xi \to 0$ is important in understanding the behavior of the coupling coefficients. In this limit

$$k_l \to \frac{\sqrt{2}}{2\pi} \int_0^{2\pi} dz a(z) \exp(-ilz), \tag{31}$$

and for very small fields, a linear relationship exists between the magnetic field and the effective field. Eq.(31) states that the coupling efficiency of the l'th harmonic at small fields is just the $\sqrt{2}$ times the complex Fourier transform of the l'th component of the vector potential normalised to unity rms value.

This small ξ formula is often useful. In particular, we have done a calculation shown in fig.(2) where the magnetic field is the difference of two delta functions. The first delta function was placed at $z = \pi/2$ and the negative delta function at $z = 3\pi/2$. In the calculation, each delta function was approximated by a top-hat distribution that was 5% of the 2π interval. Here we see that the coupling efficiency is nearly independent of ξ. Note, however, the reduction from unity of the fundamental coupling coefficient at $\xi = 0$. This reduction can be overcome by raising the overall rms value of the field as explained in fig.(1). However, if one regards the rms vector potential generated by the harmonic as restricting the vector potential on the fundamental, then this is, as we will discuss later, a difficulty that one can regard as being as serious as a reflection coefficient degraded by the coupling effiency of the fundamental.

Now we show a calculation in fig.(3) of the coupling efficiency where we have raised the efficiency of the third harmonic by considering a magnetic field where $b \propto \sin(z) - 3\sin(3z)$. Note that in this case there are important variations with ξ. At $\xi = 0$ the two coupling terms are equal as we expect from the Fourier transform rule applied to the vector potential. Remember, however, that the sum of squares of the coupling efficiencies add to unity at $\xi = 0$ and, therefore, both coupling efficiencies are reduced to $1/\sqrt{2}$. Again we are faced with the question of whether we can raise the rms magnetic field to compensate for this effect. Alternately we can ask. Can we add an arbitrary amount of the third harmonic without decreasing the fundamental? Even if we can, we are faced with the wavelength shift problem which depends on the rms field that is increasing as the third harmonic is added.

We now come to the question of the constraints on the magnetic fields. From eq.(8) we can see that a lower effective static magnetic field on the fundamental, lowers the energy loss to the electrons and decreases the gain. This decrease in gain could be made up by an increase in the reflection coefficients of the mirrors. The obtaining of good reflection coefficients on mirrors is then seen to be of nearly equivalent importance as achieving a good effective field on the fundamental. Thus, we do not want the coupling efficiency on the fundamental to drop by something on the order of 10%, because achieving 10% more reflectivity on the mirrors is a large chore. Of course, if we can increase the rms magnetic field, we will do so. To enhance the generation of higher harmonics in this study, we adopt the following criteria

$$a_{w3} \leq 0.3a_{w1},$$

and

$$a_{w5} \leq 0.3a_{w3}.$$

In fig..(4)-(6) we show the variation of the coupling efficiency with the ratios of the third to first harmonic vector potential and the fifth to first harmonic vector potential. In these calculations, the effective field is plotted and with $a_{w1} = 1.12$ these are very nearly the coupling efficiencies as defined above. The largest values of the effective field divided by those obtained from a pure sinusoidal calculation ($a_{w3} = a_{w5} = 0$) are tabulated below

Harmonic	Field	Power
3rd	1.3	1.6
5th	1.8	3.2
7th	3.7	14.
19th	62.	4×10^3

In the case of the 19th harmonic, where we have very large enhancements over the sinusoidal case, we are particularly concerned about the effect of non-uniform magnetic fields and the alignment of those fields. These phase errors will set a limit to the highest harmonic that can be usefully obtained by these techniques, and the nineteenth harmonic may be precluded therby.

One important feature of these curves is the interaction between the 3rd and 5th harmonic fields on the 7th and 19th harmonic plots, fig.(5) and (6). The fields are strongly interacting with each other. On the 19th harmonic, a stronger coupling efficiency may be obtained by simply adding a 19th harmonic component to the magnetic field, but these plots show that such measures may not be necessary. Indeed, it may turn out to be easier to manufacture the lower harmonics than the higher.

3.3. *Phase Programming*

The third way to obtain higher harmonic output is by phase programming. By phase programming we mean altering the uniformity of the wiggler so as to introduce local jumps in the electron phase. In principle, these jumps could be continuous, as would occur for a tapered wiggler, for instance. Here we consider the effect of one such jump to illustrate that benefits exist from phase programming. From eq.(8) we see that the average energy of a group of electrons, $< \gamma >$, depends on the product of $< \exp(il\phi)/\gamma >$ and $\mathcal{E}_l$. If $< \gamma >$ is dropping due to $\mathcal{E}_l$, it follows that a jump of π in the phase $l\phi$ will cause that part of $< \gamma >$ to increase. This phase jump is just π/l, and as l gets large, the effect on the fundamental diminishes. Here we report a calculation having a high intensity that shows the promise of this technique. The calculation was done at $k_w = 0.76$, and in the units reported in [1]; the dimensionless density was 3.12×10^{-6}; the dimensionless field was 2.52×10^{-4}; the Lorentz factor, γ, was 41.6; and an energy spread of 2.2% was used. In fig.(7) we show the final phase space of the electrons in the case run without the phase jump. In fig.(8) we see the magnitude of the fundamental field plotted as a function of time through the wiggler in dimensionless units. The fundamental has just reached its peak value. In fig.(9), we see the third harmonic has reached its peak and is being absorbed in the last half of the wiggler. When we introduce the phase shift of $\pi/3$ we see in fig.(10) that the fundamental is altered only slightly. The third harmonic, in fig.(11), however, no longer is absorbed and it continues to rise to a value slightly under a factor of two of its previous value. This increase corresponds to a factor of four in power.

4. Conclusion

We have shown that there are three techiques that can be used to enhance the power on XUV harmonics. The first of these is to increase the rms magnetic field to as large a value as possible. The second, is to introduce distortions to the pure sinusoidal magnetic field, thereby enhancing the coupling coefficients for the harmonics. The enhancement process is non-linear and we have shown that addition of third and fifth harmonics produce substantial coupling to the nineteenth harmonic. Enhancements at $a_{w1} = 1.12$ range from 1.6 to 4000. The enhancement on the nineteenth harmonic,however, may not all be realized because of phase errors. The third enhancement scheme is phase programming, and the case shown suggests that factors of four in power may be acheived on the third harmonic. By a combination of the three techniques, we expect order of magnitude enhancements of the harmonics.

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References

[1] C.J. Elliott and M.J. Schmitt, Proc. of Seventh International Free Electron Laser Conference (1985) Nuc. Methods in Phys. Res. 00 (1986)

[2] W.B. Colson, IEEE JQE QE-17 (1981) 1417

[3] W.B. Colson in: Physics of Quantum Electronics, vol. 5, (Addison-Wesley Publishing Co., New York,

Reading, 1978) ch. 4.

[4] J.C. Goldstein, B.E. Newman, R.K. Cooper, and J.C. Comly, Jr., Laser Techniques in The Extreme Ultraviolet (1984) AIP Conf. Proc. no. 119, Subseries on Optical and Engineering no.5, ch. 6, p.293.

[5] W.B. Colson and P. Elleaume, Appl. Phys. B. 29 (1982) 101.

Figure Captions

1 . Magnitudes of effective fields for the 1st, 3rd, 5th,... 13th harmonic, highest to lowest, respectively for a pure sinusoidal magnetic field.

2 . Magnitude of coupling efficiencies for the 1st, 3rd, ...13th harmonic with a positive deltafunction magnetic field at $z = \pi/2$ and a negative deltafunction at $z = 3\pi/2$. These deltafunctions are approximated by flat-topped distributions that are each $\pi/10$ wide. As before, the lower the harmonic, the higher the efficiency.

3 . Magnitude of coupling efficiencies for the 1st, 3rd,...13th harmonic with $b = \sin(z) - 3\sin(3z)$. The curve markers are as before.

4 . Effective field for the fifth harmonic with $a_{w1} = 1.12$, as a function of the harmonic ratios a_{w3}/a_{w1} and a_{w5}/a_{w1}. When these harmonic ratios are zero, $K_5 = 0.0898$.

5 . Effective field for the seventh harmonic. When the harmonic ratios are zero, $K_7 = 0.0378$.

6 . Effective field for the 19th harmonic. When the harmonic ratios are zero, $K_{19} = 0.00037$.

7 . Phase space plot without the phase jump.

8 . Fundamental field magnitude versus time through the wiggler when no phase jump is introduced.

9 . Third harmonic field magnitude versus tiem through wiggler when no phase jump is introduced.

1 0. Same as fig.(8) but with the phase jump. Little alteration occurs on this fundamental.

1 1. Same as fig.(9) but with the phase jump. Here the field magnitude almost doubles.

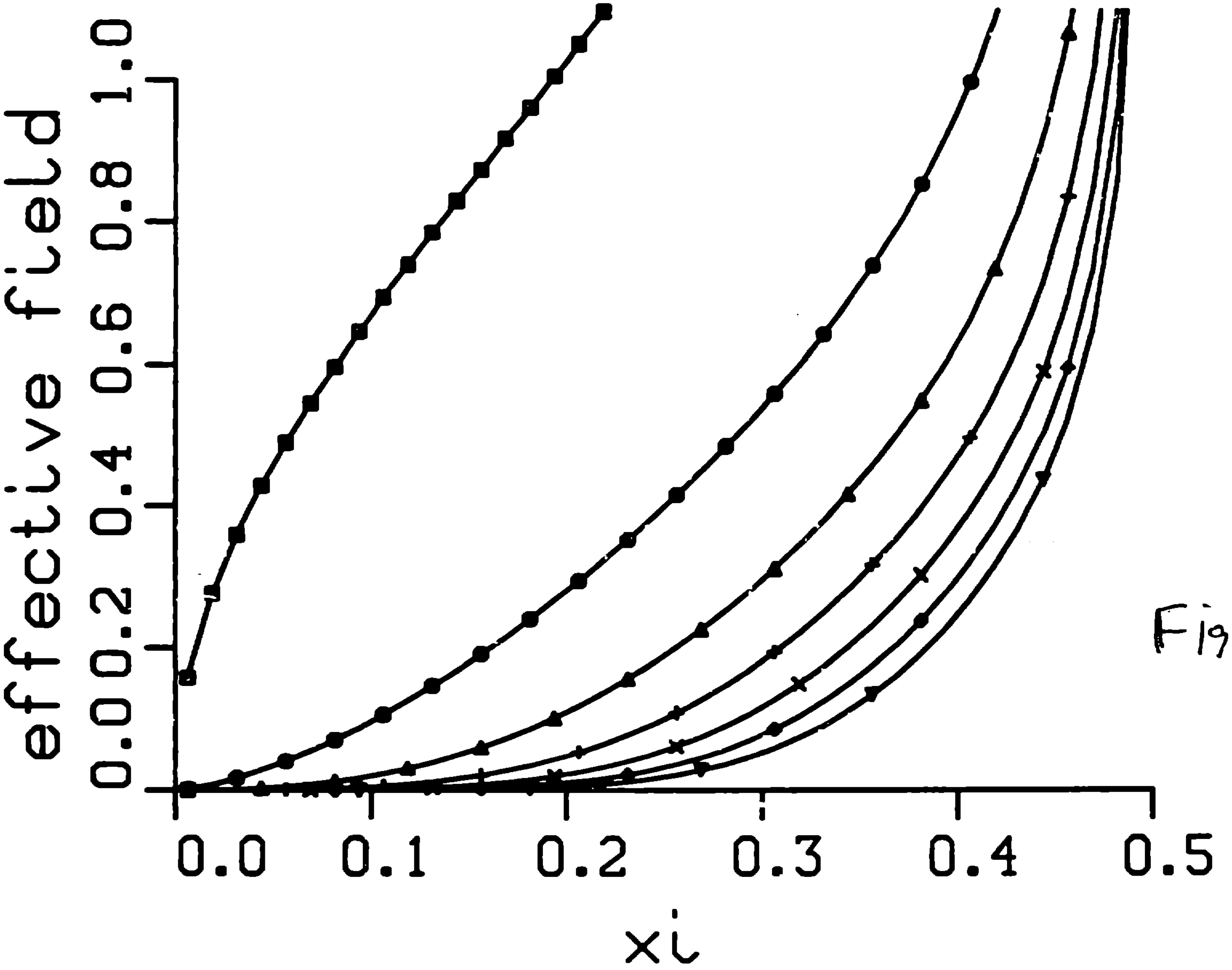


Fig. 1

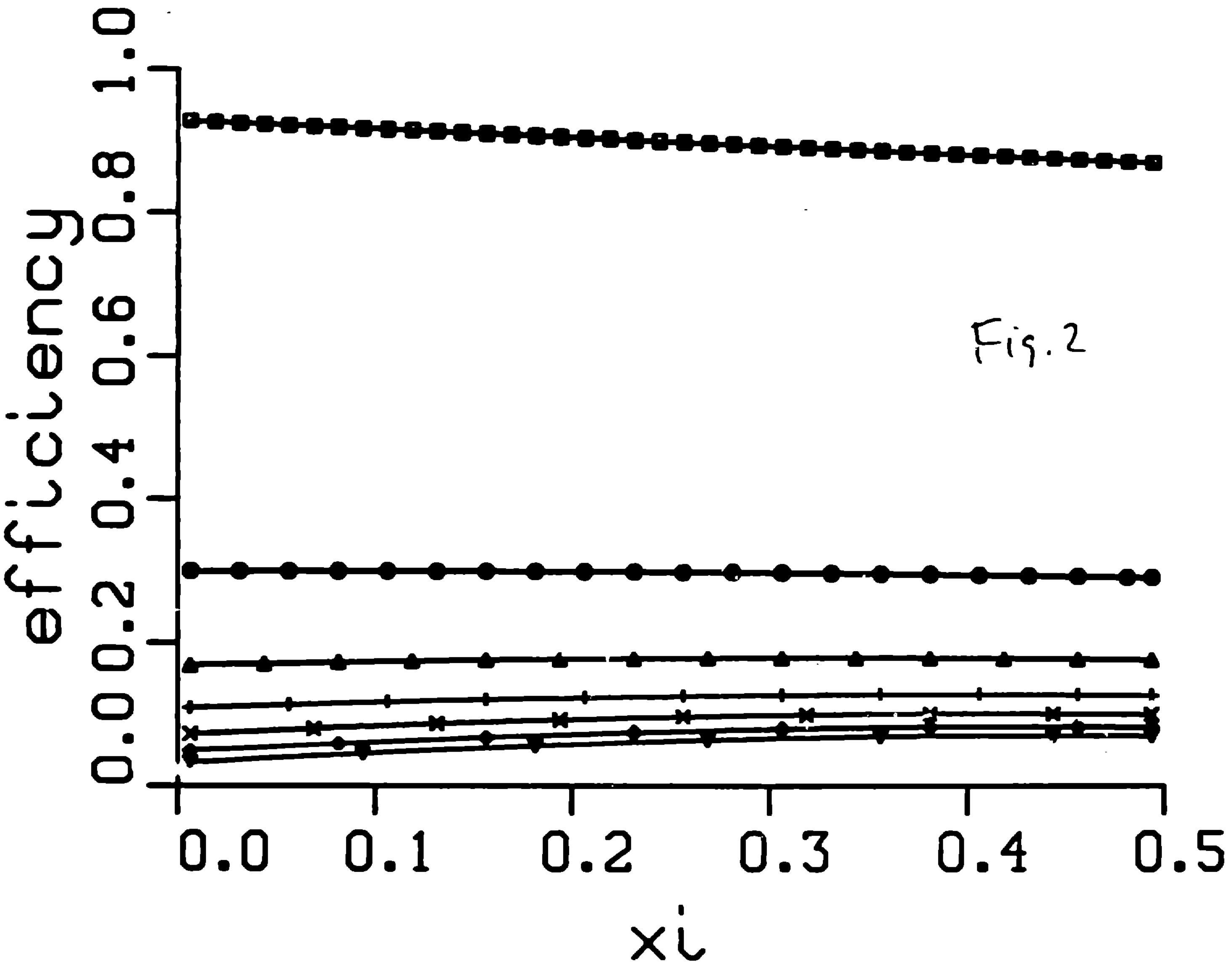
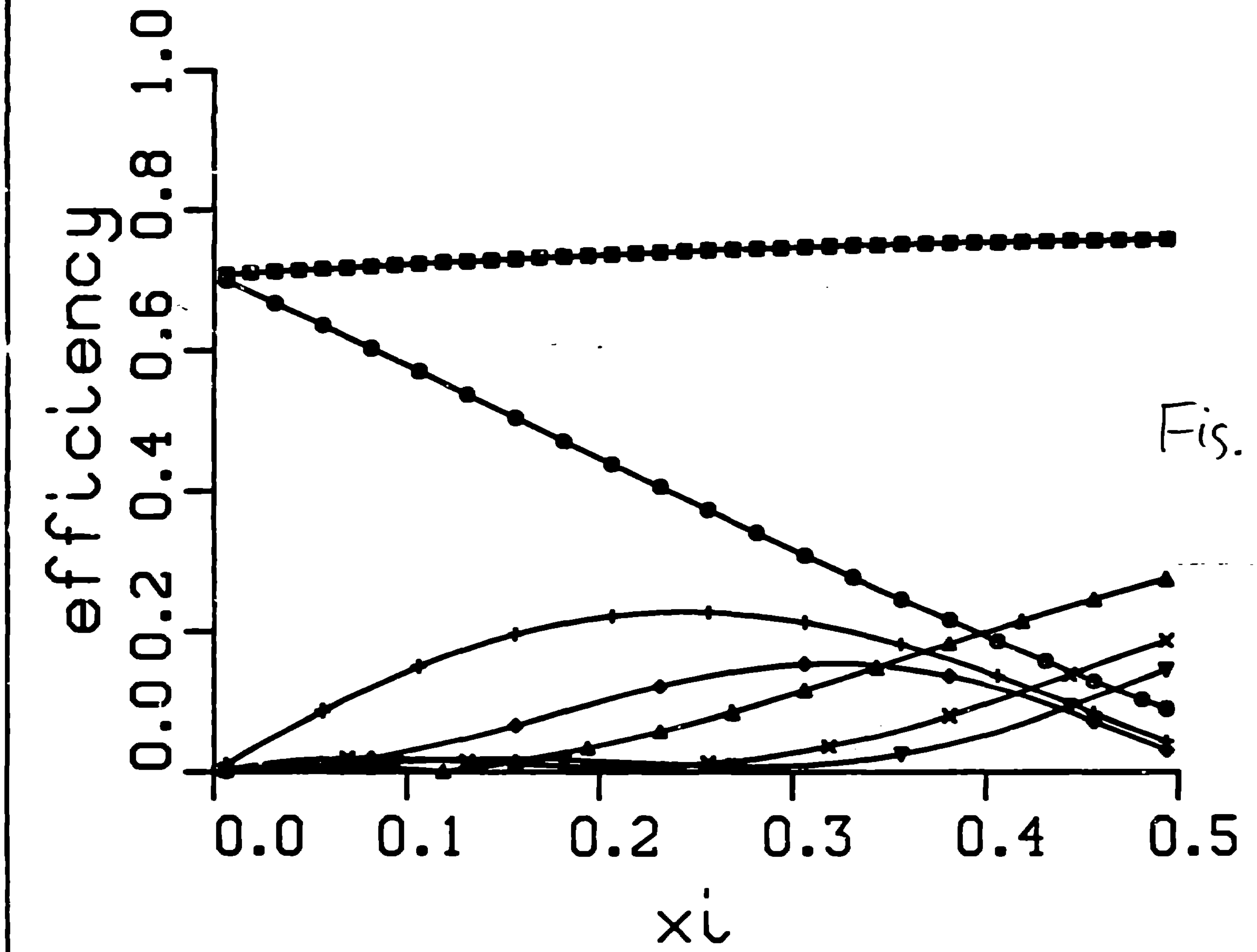


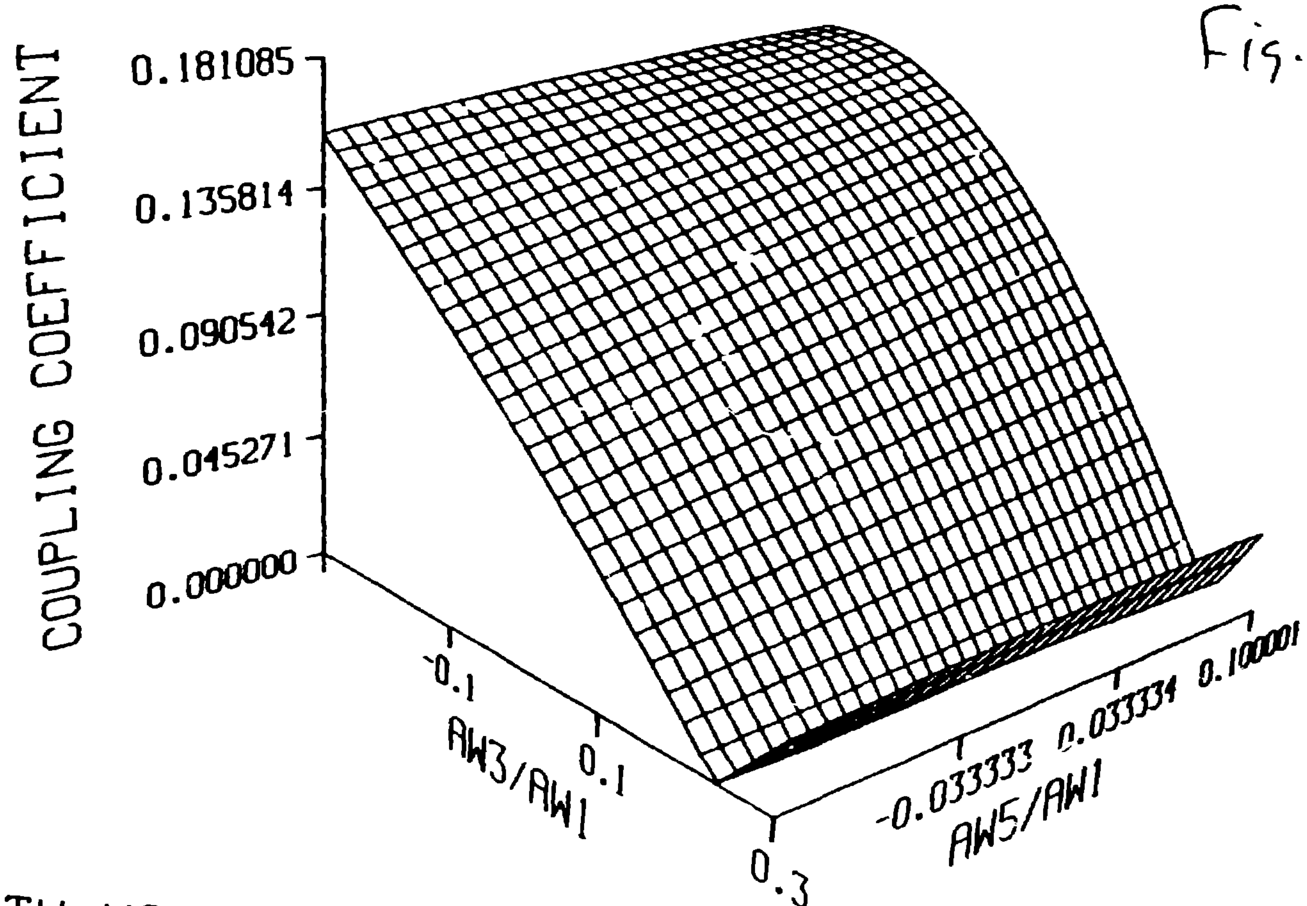
Fig. 2



Fis. 3

VARIATION WITH AW3 AND AW5

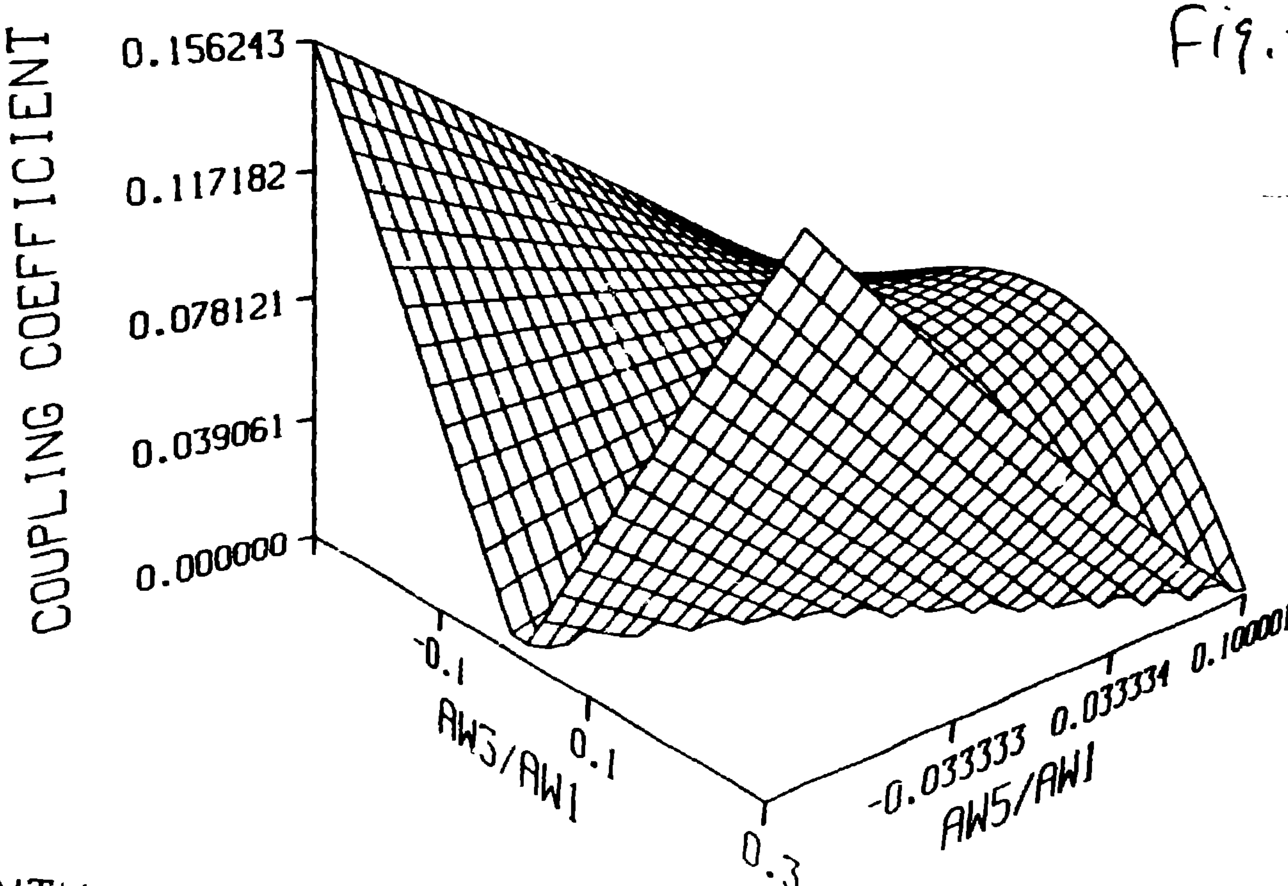
Fig. 4



FIFTH HARMONIC

VARIATION WITH AW3 AND AW5

Fig. 5



SEVENTH HARMONIC

VARIATION WITH AW3 AND AW5

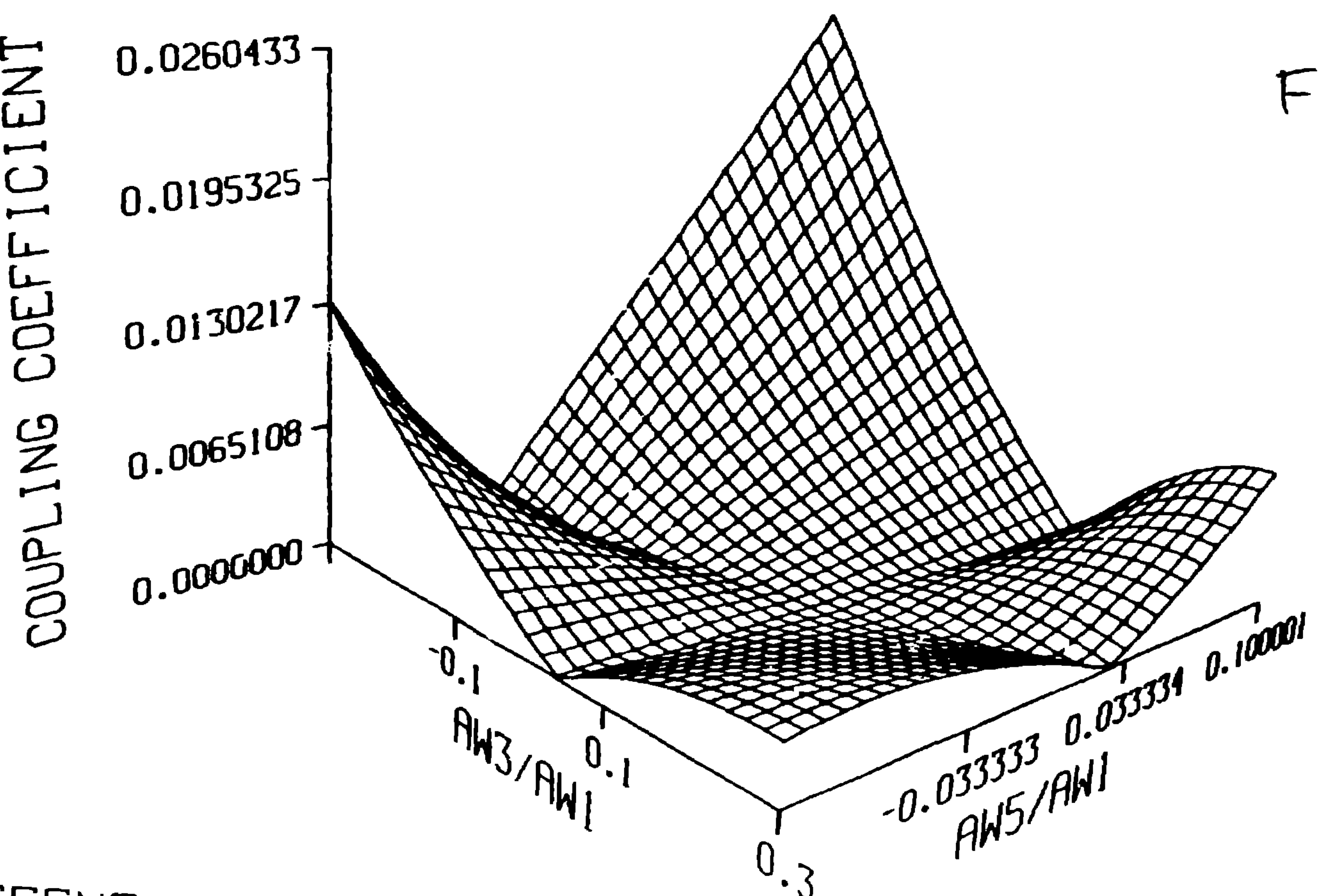


Fig. 6

NINETEENTH HARMONIC

Fig. 7

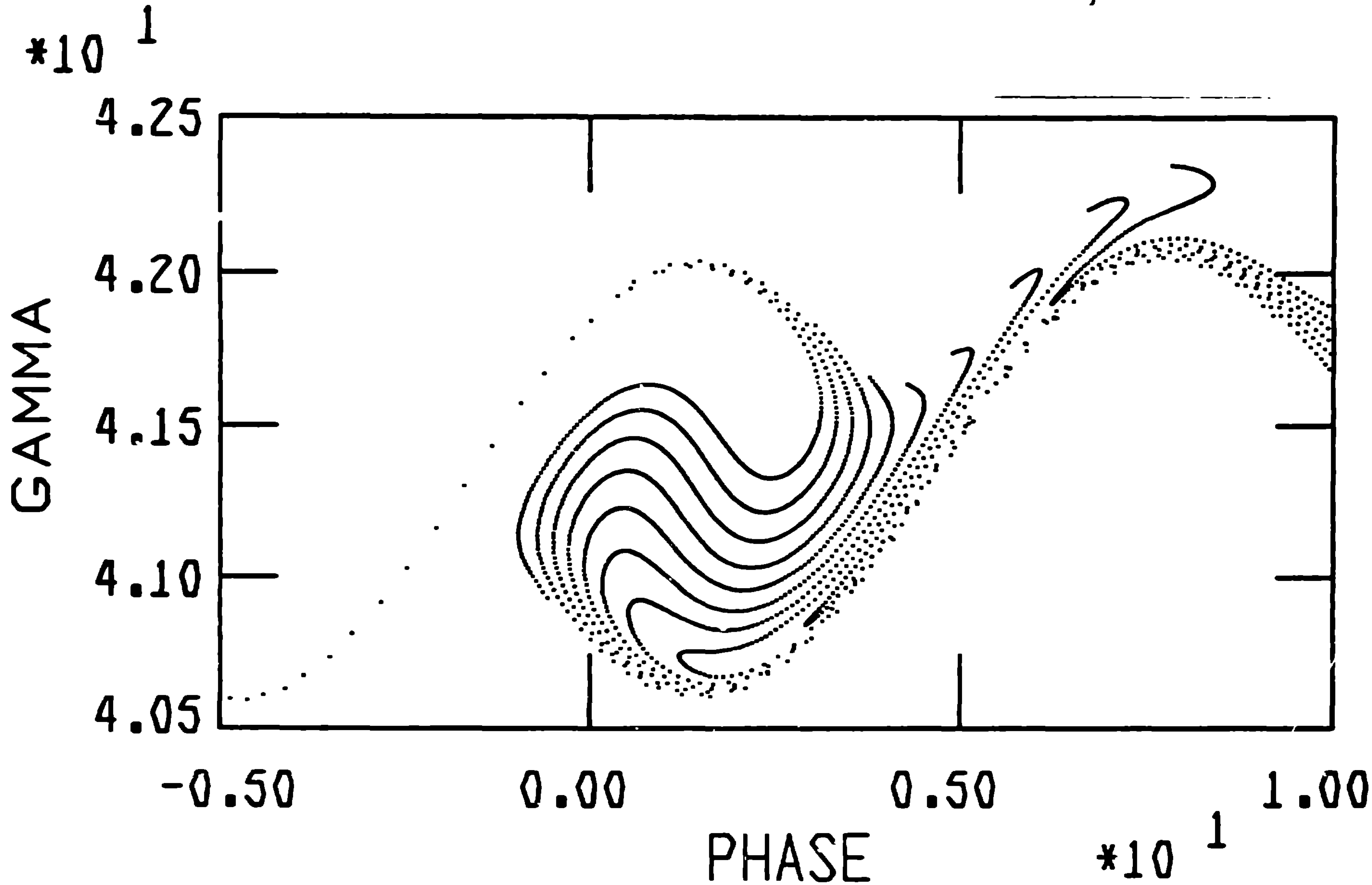


Fig. 8

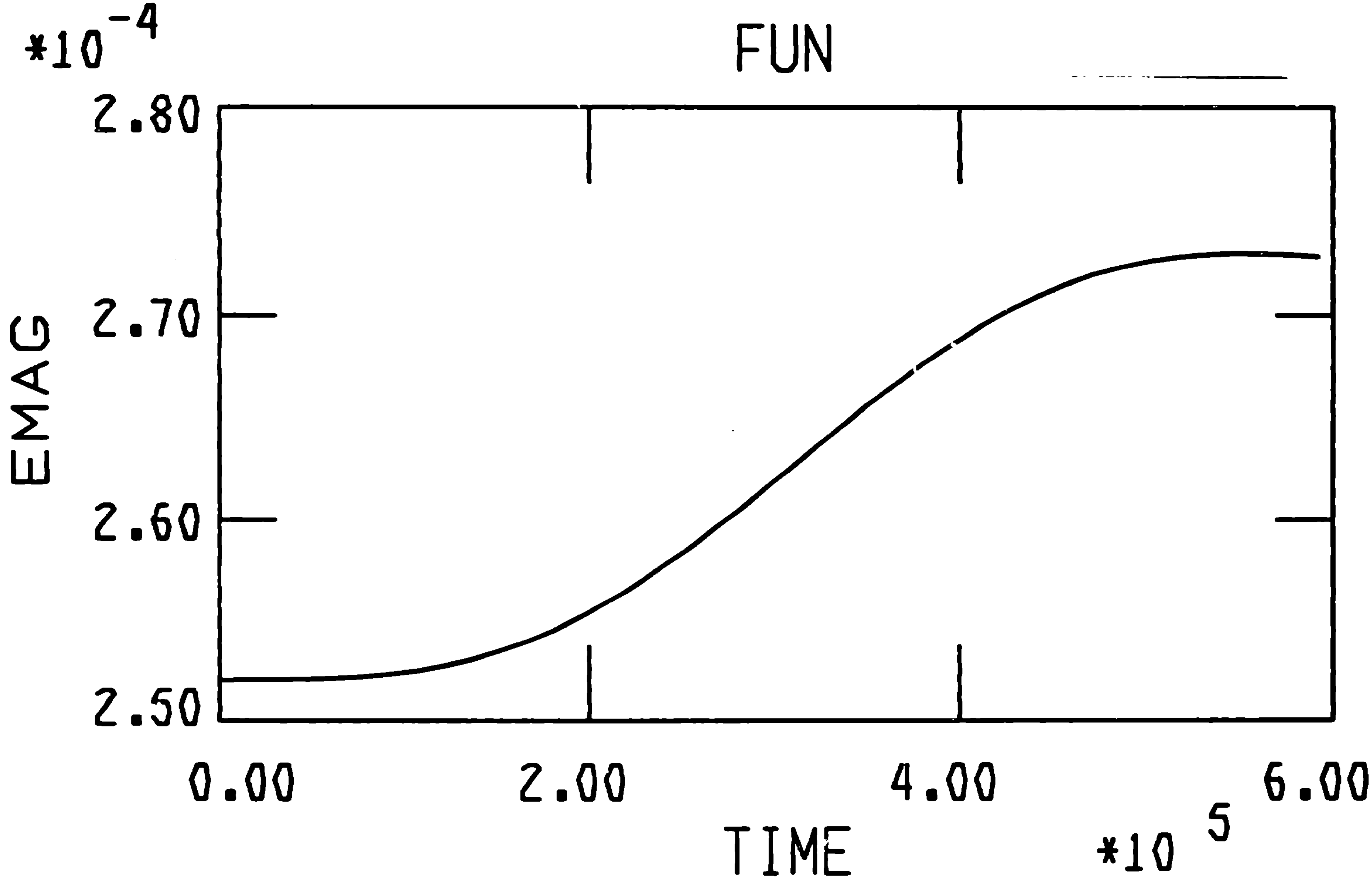


Fig. 9

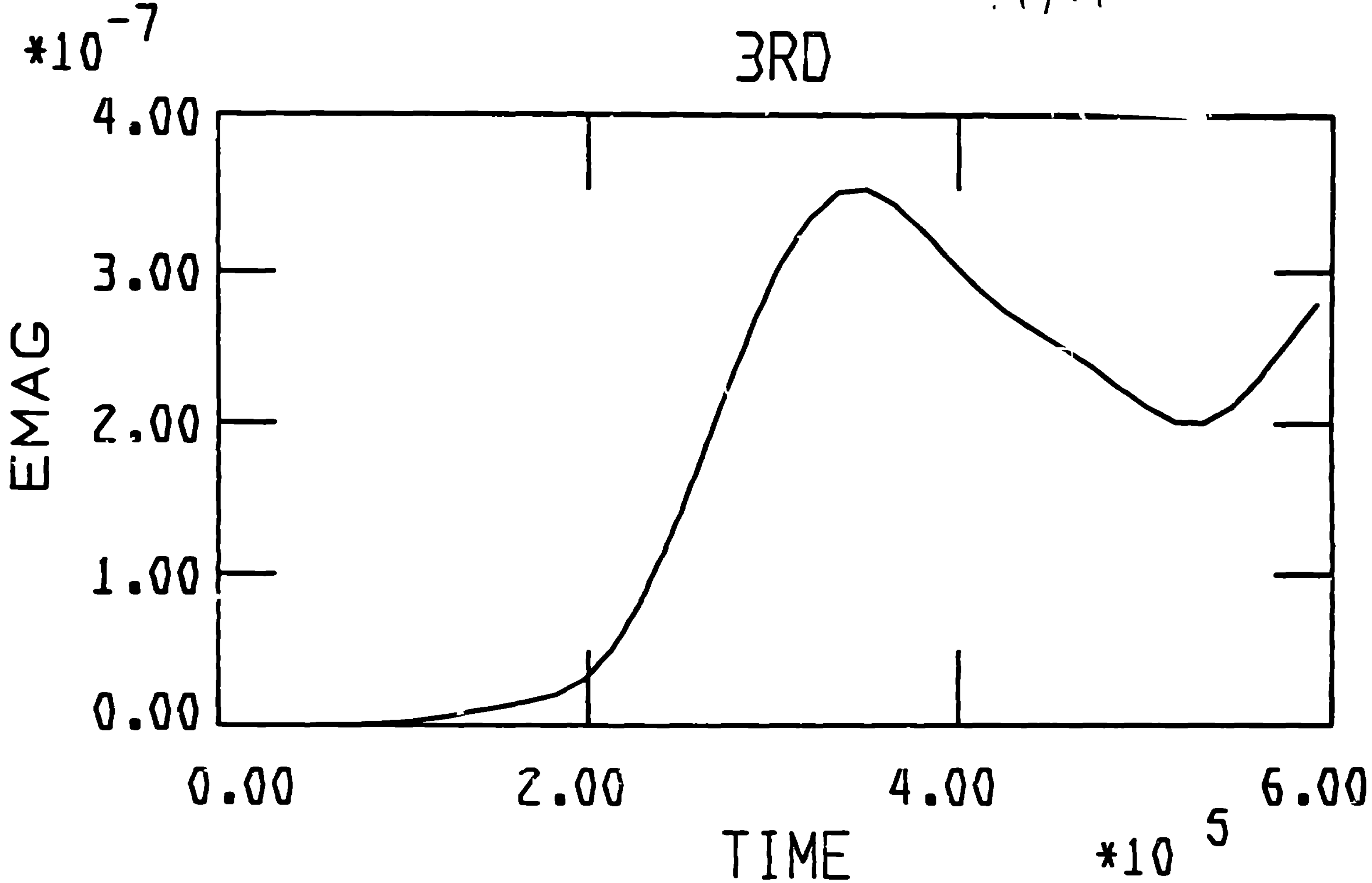


Fig. 10

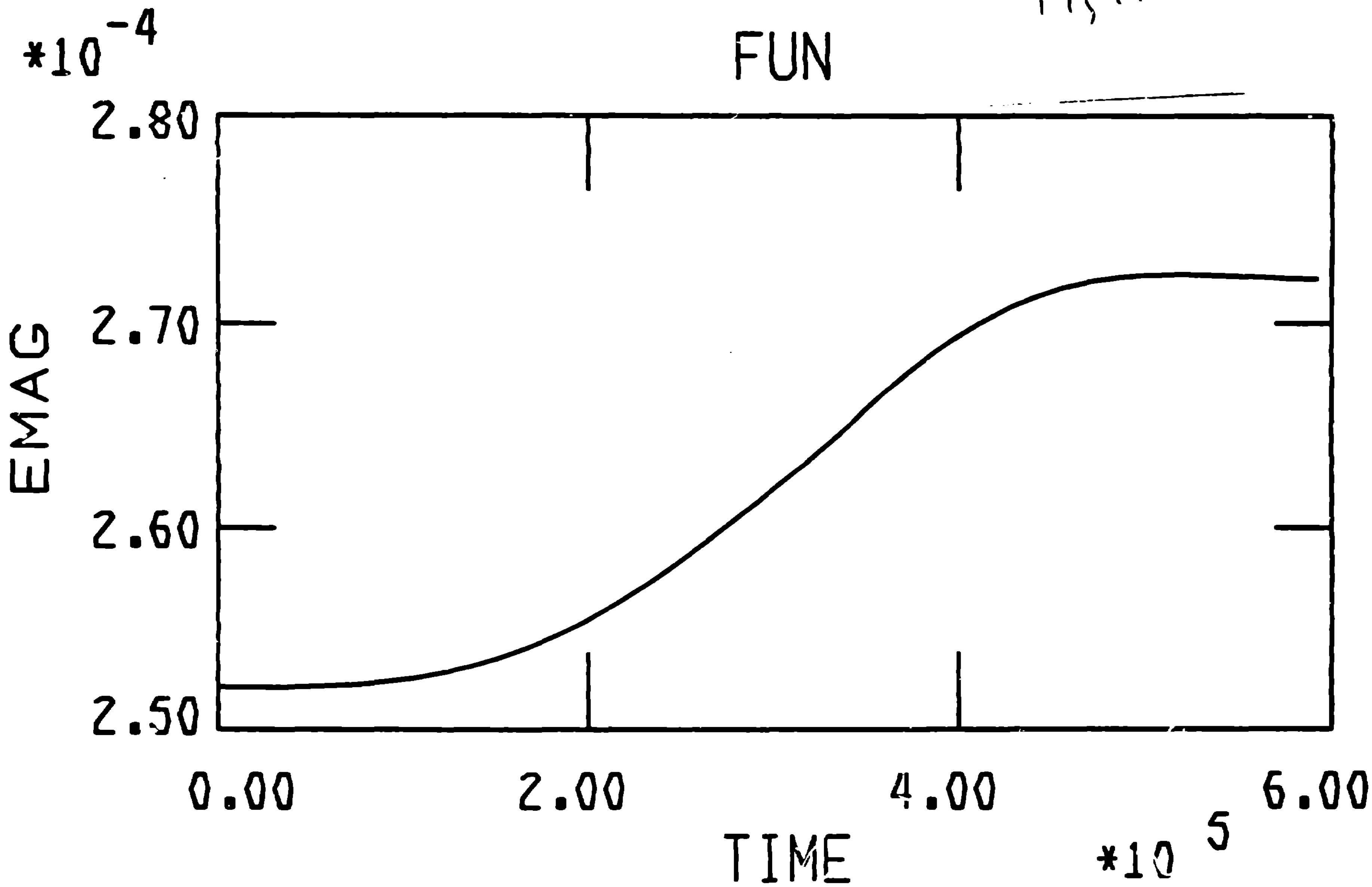


Fig. 11